Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

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10MAT21

Second Semester B.E. Degree Examination, June / July 2014
Engineering Mathematics - II

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer FIVE full questions choosing at least two from each part.***
2. Answer all objective type questions only in OMR sheet page 5 of the Answer Booklet.
3. Answers to objective type questions on sheets other than OMR will not be valued.

PART - A

1 a. Choose the correct answer : **(04 Marks)**

i) The general solution of the equation $x^2p^2 + 3xyp + 2y^2 = 0$ is ____
(A) $(y^2x - c)(xy - c) = 0$ (B) $(x-y-c)(x^2 + y^2 - c) = 0$
(C) $(xy - c)(x^2y-c) = 0$ (D) $(y-x-c)(x^2 + y^2 + c) = 0$

ii) The given differential equation is solvable for y, if it is possible to express y in terms of ____
(A) y and p (B) x and p (C) x and y (D) y and x

iii) The singular solution of Clairaut's equation is ____
(A) $y = xg(x) + f[g(x)]$ (B) $y = cx + f(c)$
(C) $cy + f(c)$ (D) $y\,g^2(x) + f[g(x)]$

iv) The singular solution of the equation $y = px - \log p$ is ____
(A) $y^2 = 4ax$ (B) $x = 1 - \log x$ (C) $y = 1 - \log\left(\frac{1}{x}\right)$ (D) $x^2 = y \log x$

b. Solve $p^2 - 2p \sinh x - 1 = 0$. **(04 Marks)**

c. Solve $y = 2px + \tan^{-1}(xp^2)$. **(06 Marks)**

d. Obtain the general solution and singular solution of Clairaut's equation is $(y - px)(p-1) = p$. **(06 Marks)**

2 a. Choose the correct answer : **(04 Marks)**

i) The complementary function of $[D^4 + 4]\,x = 0$ is ____
(A) $x = e^{-t}[c_1 \cos t + c_2 \sin t] + e^{t}[c_3 \cos t + c_4 \sin t]$
(B) $x = [c_1 \cos t + c_2 \sin t] + [c_3 \cos t + c_4 \sin t]$
(C) $x = [c_1 + c_2 t]\,e^{-t}$
(D) $x = [c_1 + c_2 t]\,e^{t}$.

ii) Find the particular integral of $(D^3 - 3D^2 + 4)\,y = e^{2x}$ is ____
(A) $\frac{x^2e^{2x}}{6}$ (B) $\frac{x^2e^{3x}}{6}$ (C) $\frac{x^2e^{x}}{6}$ (D) $\frac{x^2e^{4x}}{6}$

iii) Roots of $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$ are ____
(A) $2 \pm i$ (B) $3 \pm i$ (C) $2 \pm 2i$ (D) $-2 \pm i$

iv) Find the particular integral of $(D^3 + 4D)\,y = \sin 2x$ is ____
(A) $\frac{x \sin x}{8}$ (B) $\frac{-x \sin x}{8}$ (C) $\frac{-x \sin 2x}{8}$ (D) $\frac{x \sin 2x}{8}$

b. Solve $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = e^x + 1$. **(04 Marks)**

c. Solve $\frac{d^2y}{dx^2} - 4y = \cosh(2x - 1) + 3^x$. **(06 Marks)**

d. Solve $\frac{dy}{dx}+y=ze^{x}$, $\frac{dz}{dx}+z=y+e^{x}$. **(06 Marks)**

3 a. Choose the correct answer : **(04 Marks)**

i) The Wronskian of x and xe^{x} is _____

(A) e^{x} (B) e^{2x} (C) e^{-2x} (D) e^{-x}.

ii) The complementary function of $x^2y'' - xy' - 3y = x^2 \log x$ is _____

(A) $c_1 \cos(\log x) + c_2 \sin(\log x)$ (B) $c_1 x^{-1} + c_2x^3$.

(C) $c_1x + c_2x^3$ (D) $c_1\cos x + c_2 \sin x$.

iii) To transform $(1+x)^2y'' + (1+x)y' + y = 2 \sin \log(1+x)$ into a linear differential equation with constant coefficient _____

(A) $(1+x)=e^{t}$ (B) $(1+x)=e^{-t}$ (C) $(1+x)^2=e^{t}$ (D) $(1-x)^2=e^{t}$.

iv) The equation $a_0(ax+b)^2y'' + a_1(ax+b)y' + a_2y = \phi(x)$ is _____

(A) Simultaneous equation (B) Cauchy's linear equation

(C) Legendre linear equation (D) Euler's equation.

b. Using the variation of parameters method to solve the equation $y'' + 2y' + y = e^{-x} \log x$. **(04 Marks)**

c. Solve $x^2\frac{d^2y}{dx^2}-(2m-1)x\frac{dy}{dx}+(m^2+n^2)y = n^2x^m \log x$. **(06 Marks)**

d. Obtain the Frobenius method solve the equation

$x\frac{d^2y}{dx^2}+\frac{dy}{dx}-y=0$. **(06 Marks)**

4 a. Choose the correct answer : **(04 Marks)**

i) Partial differential equation by eliminating a and b from the relation $Z=(x-a)^2+(y-b)^2$ is _____

(A) $p^2q^2=4z$ (B) $pq=4z$ (C) $r=4z$ (D) $t=4$

ii) The Lagranges's linear partial differential equation $Pp + Qq = R$ the subsidiary equation is _____

(A) $\frac{dx}{R}=\frac{dy}{P}=\frac{dz}{Q}$ (B) $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$ (C) $\frac{dx}{Q}=\frac{dy}{R}=\frac{dz}{P}$ (D) $\frac{dx}{P}+\frac{dy}{Q}+\frac{dz}{R}$

iii) By the method of separation of variable we seek a solution in the form is_____

(A) $x=x+y$ (B) $z=x^2+y^2$ (C) $x=z+y$ (D) $x=x(x)\,y(y)$

iv) The solution of $\frac{\partial^2 z}{\partial x^2}=\sin(xy)$ is _____

(A) $z=-x^2\sin(xy)+y\,f(x)+\phi(x)$ (B) $\frac{-\sin(xy)}{y^2}+x\,f(y)+\phi(y)$

(C) $z=\frac{-\sin xy}{x^2}+y\,f(x)+\phi(x)$ (D) None of these.

b. Form the partial differential equation of all sphere of radius 3 units having their centre in the xy - plane. **(04 Marks)**

c. Solve $x(y^2+z)p - y(x^2+z)q = z(x^2-y^2)$. **(06 Marks)**

d. Use the method of separation of variables to solve

$y^3\frac{\partial z}{\partial x}+x^2\frac{\partial z}{\partial y}=0$. **(06 Marks)**

PART - B

5 a. Choose the correct answer : **(04 Marks)**

i) The value of $\int_0^1\int_0^{x^2} e^{y/x}\,dydx$ is _____

(A) 0 (B) 1 (C) 3 (D) ½ .

ii) The value of $\Gamma(\frac{1}{2})$ is ____

(A) $2\sqrt{\pi}$ (B) $\pi\sqrt{2}$ (C) $\sqrt{\pi}$ (D) $\sqrt{2\pi}$.

iii) The integral $\int_0^a \int_y^a \frac{x}{x^2+y^2}\,dxdy$ after changing the order of integration is _____

(A) $\int_0^a \int_0^x \frac{x}{x^2+y^2}\,dydx$ (B) $\int_0^a \int_0^x \frac{x}{x^2+y^2}\,dxdy$

(C) $\int_0^x \int_0^x \frac{x}{x^2+y^2}\,dxdy$ (D) $\int_0^x \int_0^a \frac{x}{x^2+y^2}\,dxdy$

iv) The value of $\beta(3, \frac{1}{2})$ is _____

(A) $\frac{15}{16}$ (B) $\frac{16}{15}$ (C) $\frac{16}{5}$ (D) $\frac{16}{3}$

b. Change the order of integration in $\int_0^{4a} \int_{\frac{x^2}{4a}}^{\sqrt{ax}} dydx$ and hence evaluate the same. **(04 Marks)**

c. Evaluate $\int_{-c}^{c} \int_{-b}^{b} \int_{-a}^{a} (x^2+y^2+z^2)\,dx\,dy\,dz$. **(06 Marks)**

d. Prove that $\int_0^1 \frac{x^2}{\sqrt{1-x^4}}dx \times \int_0^1 \frac{1}{\sqrt{1+x^4}}dx = \frac{\pi}{4\sqrt{2}}$. **(06 Marks)**

6 a. Choose the correct answer : **(04 Marks)**

i) Let S be the closed boundary surface of a region of volume V then for a vector field f defined in V and in S $\int_C f.nds$ is ________

(A) $\int_V \text{curl}\, f\, dv$ (B) $\int_V \text{div}\, f\, dv$ (C) $\int_V \text{grad}\, f\, dv$ (D) None of these

ii) If $\int_C f.dr$ where $f = 3xy\hat{i} - y^2\hat{j}$ and C is the part of the parabola $y = 2x^2$ from the region (0, 0) to the point (1, 2) is__________

(A) $7/6$ (B) $-7/6$ (C) $3x + 3y$ (D) -35

iii) In the Green's theorem in the plane $\oint_C Mdx + Ndy =$ __________

(A) $\iint_R \left(\frac{\partial M}{\partial y} + \frac{\partial N}{\partial x}\right)dxdy$ (B) $\iint_R \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)dxdy$

(C) $\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)dxdy$ (D) $\iint_R \left(\frac{\partial N}{\partial x} + \frac{\partial M}{\partial y}\right)dxdy$

iv) A necessary and sufficient condition that the line integral $\int_C \vec{F}\cdot d\vec{r}$ for any closed curve C is _______________.

(A) $\text{div}\,\vec{F} = 0$ (B) $\text{div}\,\vec{F} \neq 0$ (C) $\text{curl}\,\vec{F} = 0$ (D) $\text{grad}\,\vec{F} = 0$

b. Using the divergence theorem, evaluate $\int_S f\cdot nds$ where $f = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and S is the surface of the cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0, z = 1. **(04 Marks)**

c. Use the Green's theorem, evaluate $\iint_C (2x^2 - y^2)dx + (x^2 + y^2)dy$ where C is the triangle formed by the lines x = 0, y = 0 and x + y = 1. **(06 Marks)**

d. Verify the Stoke's theorem for $f = -y^3\hat{i} + x^3\hat{j}$ where S is the circle disc $x^2 + y^2 \leq 1, z = 0$. **(06 Marks)**

7 a. Choose the correct answer : **(04 Marks)**

i) $L\{\sinh at\} =$ ____________

(A) $\frac{s}{s^2-a^2}$ (B) $\frac{s}{s^2+a^2}$ (C) $\frac{a}{s^2-a^2}$ (D) $\frac{a}{s^2+a^2}$.

ii) If $L\{f(t)\}=F(s)$ then $L\{e^{at}f(t)\}$ is __________

(A) F(s+a) (B) F(s-a) (C) F(s) (D) None of these

iii) $L\left\{\frac{e^t \sin t}{t}\right\}$

(A) $\frac{\pi}{2}+\tan^{-1}(s-1)$ (B) $\frac{\pi}{2}+\tan^{-1}s$ (C) $\frac{\pi}{2}-\cot^{-1}s$ (D) $\cot^{-1}(s-1)$

iv) Transform of unit step function $L\{u(t-a)\}$ is, _________

(A) $\frac{e^{as}}{s}$ (B) $\frac{e^{-s}}{s}$ (C) $\frac{e^{2s}}{s}$ (D) $\frac{e^{-as}}{s}$

b. Evaluate $L\left\{3^t+\frac{\cos 2t-\cos 3t}{t}+t\sin t\right\}$. **(04 Marks)**

c. Find the Laplace transform of the triangular wave, given by, $f(t)=\begin{cases} t & 0<t<C \\ 2C-t & C<t<2C \end{cases}$ and $f(t+2c)=f(t)$. **(06 Marks)**

d. Express $f(t)=\begin{cases} \cos t & \text{if } 0<t<\pi \\ \cos 2t & \text{if } \pi<t<2\pi \\ \cos 3t & \text{if } t>2\pi \end{cases}$ in terms of unit step function and hence find $L\{f(t)\}$. **(06 Marks)**

8 a. Choose the correct answer : **(04 Marks)**

i) $L^{-1}\left\{\cot^{-1}\left(\frac{s}{a}\right)\right\}=$ ___________

(A) $\frac{\sin t}{t}$ (B) $\frac{\sin at}{t}$ (C) $\frac{\sinh at}{t}$ (D) $\frac{\sinh t}{t}$

ii) $L^{-1}\left\{\frac{1}{4s^2-36}\right\}=$ ___________

(A) $\frac{\cosh 6t}{4}$ (B) $\frac{\sin 3t}{12}$ (C) $\frac{\sinh 3t}{12}$ (D) $\frac{\cosh 3t}{6}$

iii) $L^{-1}\left\{\frac{1}{s(s^2+a^2)}\right\}=$ ___________

(A) $\frac{1-\cos at}{a^2}$ (B) $\frac{1+\cos at}{a^2}$ (C) $\frac{1-\sin at}{a^2}$ (D) $\frac{1+\sin at}{a^2}$

iv) $L^{-1}\left\{\frac{s^2-3s+4}{s^4}\right\}=$ ___________

(A) $1-3t+2t^3$ (B) $1+\frac{t^2}{3}$ (C) $t+\frac{3}{2}t^2+1$ (D) $t-\frac{3}{2}t^2+\frac{2}{3}t^3$

b. Find $L^{-1}\left\{\frac{3s+7}{s^2-2s-3}\right\}$. **(04 Marks)**

c. Using Convolution theorem evaluate $L^{-1}\left\{\frac{1}{(s+1)(s^2+4)}\right\}$. **(06 Marks)**

d. Solve $\frac{d^2y}{dt^2}+5\frac{dy}{dt}+6y=5e^{2t}$ given that $y(0)=2$, $\frac{dy(0)}{dt}=1$ by using Laplace transform method. **(06 Marks)**

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